Reply to Comment by Paul R. Motyka and G. Warren Hall

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WE wish to thank P. R. Motyka and G. W. Hall for their comments[1] on our decoupling paper.[2] They have correctly explained why we arrived at a result different from theirs when solving their problem.[3] When the calculations presented in Ref. 2 were done, we were at a loss to explain the nonagreement of results. It was discovered that their solution did not give the prescribed eigenvalues and we incorrectly assumed that was the explanation. As Motyka and Hall[1] have correctly pointed out, we had, in fact, assigned the eigenvalues to different eigenvectors.

It is not true, however, that this is an inherent shortcoming of the geometric approach. As was pointed out,[2] the geometric theory provides a method for placing the poles that was not used in Ref. 2. In fact, for the lateral motions case, it is a simple matter to assign eigenvalues independently to the individual controllability subspaces.[4] It is true that the heuristic Penalty Function Method described in Ref. 2 must be used with considerable care, and we are indebted to Motyka and Hall for making this point.

Next, we consider the comments relating to "exact" model following. In order to minimize confusion, we shall try to be somewhat precise. In the notation of Ref. 3, the given system is described by

$$\dot{x} = Fx + Gu \tag{1}$$

while the model obeys

$$\dot{x}_m = F_m x_m + G_m u_m \tag{2}$$

Since it is desired that the system behave like the model, we equate the right sides of Eq. (1) and (2), put $x_m = x$, and for fixed x and u_m, "solve" for u. Hall and Motyka[3] give

$$u = (G^T G)^{-1} G^T [(F_m - F)x + G_m u_m]$$

as the "solution." As pointed out in Ref. 2, such u minimizes‡ $||\dot{x} - \dot{x}_m||$, but need not make the difference zero. If one substitutes the above value of u into Eq. (1), it can be shown that

$$\dot{x} - \dot{x}_m = [I - G(G^T G)^{-1} G^T][(F_m - F)x + G_m u_m].$$

For the general structure of G, given in Ref. 3, it is found that the first term in the above product is the matrix $[\theta \; e_2 \; \theta \; \theta]$ (where $e_2 = [0\ 1\ 0\ 0]^T$ and $\theta = [0\ 0\ 0\ 0]^T$). It is clear then that $\dot{x} = \dot{x}_m$ only if the second row of $[(F_m - F)x + G_m u_m]$ is zero. Since the model has $\phi_m = p_m$, it is necessary that $\phi = p$. Of course, if $\phi \neq p$, it is possible to hypothesize a model for which $\dot{x} = \dot{x}_m$. In this sense, it was, perhaps, a bit strong to say that the method fails. However, it is hoped that the above discussion clarifies our point. More complete discussions of the implicit model following method may be found in Refs. 6 and 7.

Finally, in order that the reader may be able to judge the relative merits of the various approaches to decoupling, we point out that for many problems it is not possible to decouple and stabilize a system using state feedback only. It has been shown[8] that if one attempts to decouple speed and flight path angle in the longitudinal motions of an aircraft using state feedback alone, then except in contrived cases, there will be a pair of fixed eigenvalues. In order to alleviate this difficulty, one must use dynamics in the feedback controller. This problem is treated in a natural way in the geometric theory,[9,10] whereas we suspect it would be generally difficult to formulate and solve by model following methods.

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‡Under the assumption that G has full rank, the quantity $(G^T G)^{-1} G^T$ is the *generalized inverse* of G[5].

References

[1]Motyka, P. R. and Hall, G. W. "Comparison of Geometric and Response-Feedback Approaches to Aircraft Lateral-Directional Decoupling," *Journal of Aircraft*, Vol. 10, No. 11, Nov. 1973, pp.

[2]Cliff, E. M. and Lutze, F. H. "Application of Geometric Decoupling Theory to Synthesis of Aircraft Lateral Control Systems," *Journal of Aircraft*, Vol. 9, No. 11, Nov. 1972, pp. 770–776.

[3]Hall, G. W. "*A Flight Test Investigation of Direct Side Force Control*," AFFDL-TR-71-106, Sept. 1971, Air Force Flight Dynamics Lab., Wright-Patterson Air Force Base, Ohio.

[4]Wonham, W. M. and Morse, A. S. "Decoupling and Pole Assignment in Linear Multivariable Systems: A Geometric Approach," *SIAM Journal on Control*, Vol. 8, Feb. 1970, pp. 1–18.

[5]Penrose, R. "A Generalized Inverse for Matrices," *Proceedings of the Cambridge Philosophical Society*, Vol. 51, 1955, pp. 406–413.

[6]Erzberger, H. "On the Use of Algebraic Methods in the Analysis and Design of Model Following Control Systems," TN D4663, July 1968, NASA.

[7]Morse, A. S. "Structure and Design of Linear Model Following Systems," *Proceedings Joint Automatic Control Conference*, Vol. 14, 1973, pp. 263–268.

[8]Cliff, E. M. and Lutze, F. H. "Decoupling Longitudinal Motions of an Aircraft," *Proceedings Joint Automatic Control Conference*, Vol. 14, 1973, pp. 86–91.

[9]Morse, A. S. and Wonham, W. M. "Decoupling and Pole Assignment by Dynamic Compensation," *SIAM Journal on Control*, Vol. 8, August 1970.

[10]Morse, A. S. and Wonham, W. M. "Status of Noninteracting Control," *IEEE Transactions on Automatic Control*, Vol. AC-16, Dec. 1971, pp. 568–581.

Comments on "Low-Area Ratio, Thrust-Augmenting Ejectors"

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FANCHER'S paper[1] contains errors and omissions which render his conclusions rather ambiguous, at best. The following observations are offered in support of this contention. Fancher's notation is retained as far as possible, and new equations which parallel those in Ref. 1 are indicated by means of barred numbers.

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